Angle Random Walk

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Angle random walk is commonly used as a specification for rate sensors. What is it?

An angular rate sensor measures the rotation rate about its sensitive axis. The output of a rate sensor will be a signal proportional to deg/sec. Noise is often thought of as the short-term variation in the output, such as the peak-to-peak output variation or the standard deviation of the output while the sensor is at rest. These measures of noise will also be in units of deg/sec. Noise can also be defined as a function of frequency using a power spectral density (PSD) or Fast Fourier Transform (FFT). In these cases, the noise specification will be a noise density in deg/sec/Hz$^{1/2}$ or (deg/sec)^2/Hz that describes the output noise as a function of the bandwidth of the sensors. Often one is interested in using the rate sensor to track *angle* changes, not simply angular rate. In this case, the output of the angular rate sensor is integrated over time to find the angle as a function of time. Figure 1 shows the output of a rate sensor turned through 360 deg, and the calculated angle from this data. For this application, the noise specification is a bit harder to understand. How does angular rate noise affect an angle calculation?

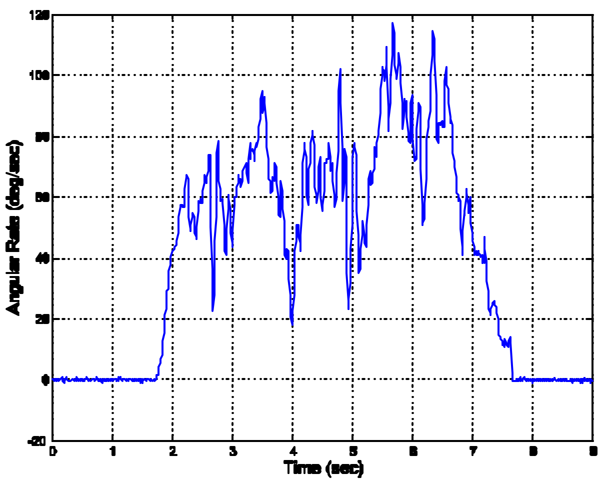

Figure 1a. Output from an angular rate sensor turned 360 deg about its sensitive axis.

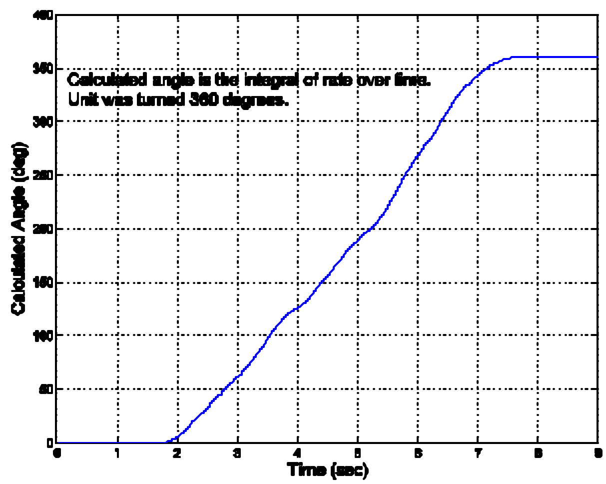

Figure 1b. Angle as a function of time found by integrating the angular rate over time.

Angle random walk (ARW) is a noise specification, in units of deg/hr$^{1/2}$, that is directly applicable to angle calculations. ARW describes the average deviation or error that will occur when you integrate the signal. This is error occurring specifically because of noise in the rate signal, independent of other characteristics that contribute to angle error (such as scale factor error or bias error.) This error will increase the longer you integrate, and provides a fundamental limitation to any angle measurement that relies solely on integration of rate.

The "Random walk" is used in statistics to describe a situation where the output of a system is driven by random, uncorrelated "steps". For example, imagine someone flipping a coin. A perfect coin has equal chances of landing heads or landing tails. Let's put this person on the 50 yard line of a football field. Every time he flips the coin, he takes one step forward when it lands heads, and one step backwards when it lands tails. The output of this system is his final position, and the input is the steps in a random direction driven by the flip of the coin. After 1000 flips of the coin, where will he be? (Just to make things easy, we'll say one step equals one yard.) The best guess would be right on the 50 yard line – since we would guess he would get an equal number of heads and tails. But he could be as far as 1000 yards away (all heads or all tails, extremely unlikely), or somewhere in between. In fact, if he did this experiment over and over, he would start to build a distribution of end points. Figure 2 shows an example distribution of 1000 trials of 1000 coin flips. Most of the trials ended near the start, but a significant number ended up more than 30 yards from the start. The shape of the distribution looks like the classical "bell curve".

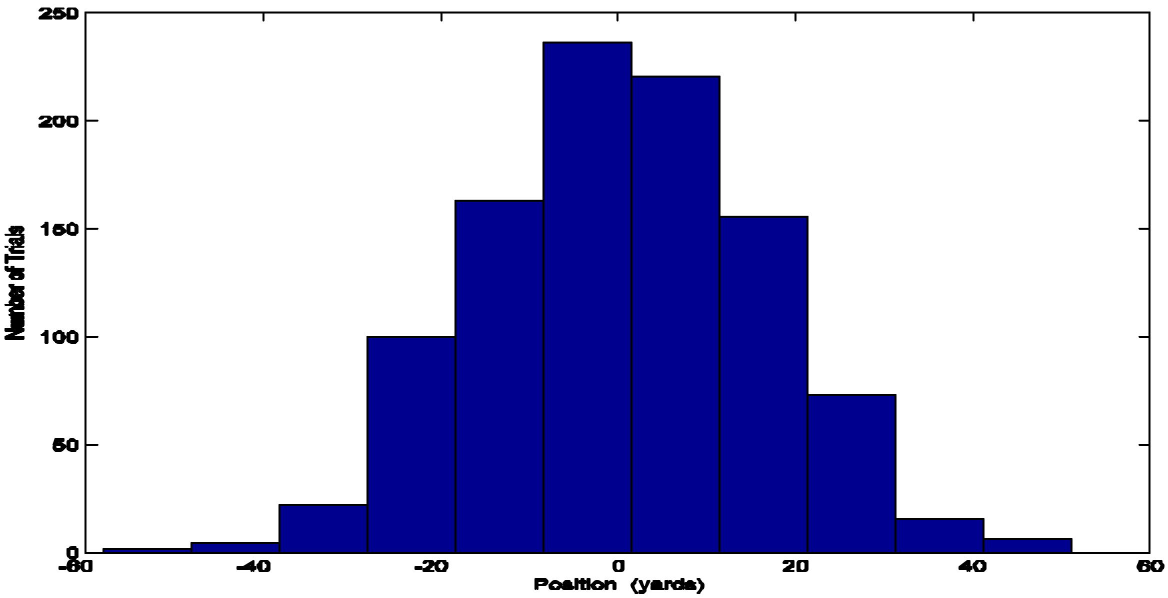

Figure 2. Distribution of results of 1000 trials of 1000 coin flips each

The standard deviation of this distribution is 15.6. This means that about 68% of the time, after 1000 flips, our coin flipper is within 15.6 yards of the start. The standard deviation will about ½ the square root of the number of flips. The more times he flips the coin, the wider the distribution will be. If he flipped the coin once per second, then the distribution would scale as the square root of the time elapsed. I.e., a longer time means more coin flips means more chance of wandering away from the start.

The output of a rate sensor is similar to the flipping coin. With the sensor at rest, the output should be zero, but there is always noise added on. (The output might have a bias value, but we can measure this and subtract it out.) Sometimes the noise will take the output above zero, and sometimes below. On average (for white noise) the time above and below zero will even out. If we integrate the signal to get angle, we would hope to have the integration equal zero, since we know the sensor is at rest. The noise, however, will make the integration bounce around. Just like flipping a coin, when the noise is above zero, the integration will "step" up; when the noise is below zero, the integration will "step" down. Again, we will get a distribution of end points. The standard deviation of the distribution will scale linearly with the noise level (the step size) and with the square root of time (the number of flips.) Even in this case, when we know the sensor is at rest, the noise adds a random component to the angle calculation that cannot be predicted or corrected for. Figure 3 shows the result of integrating 1000 noisy signals over 1000 sec.

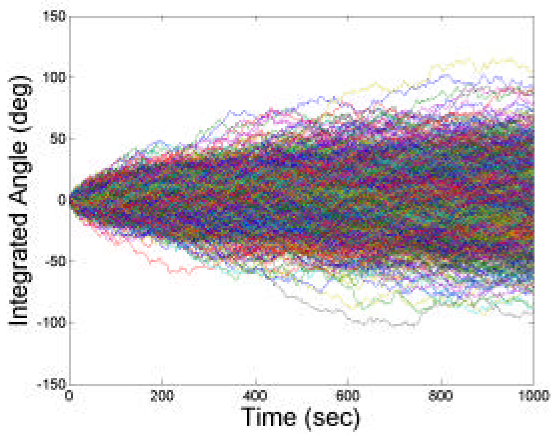

Figure 3a. 1000 trials, integrating a noise rate sensor for 1000

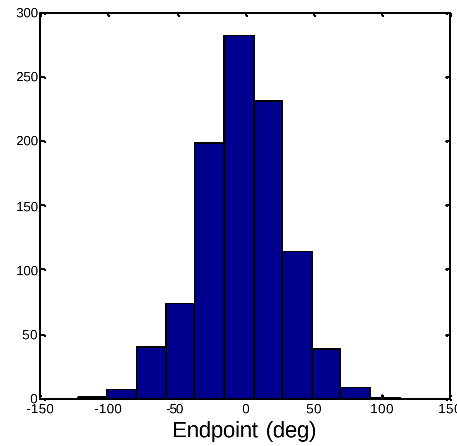

Figure 3b. The distribution of endpoints for the 1000 trials.

We can clearly see the same characteristics of the random walk that we saw on the football field. Figure 3 on the left shows how the distribution widens with time, and on the right how the distribution falls into a bell curve.

We can make a quantitative statement about the distribution of the end points for this sensor. The sensor has an ARW of 0.99 deg/sec$^{1/2}$. So after 1 second, the standard deviation of the distribution will be about 1 degree; after 100 seconds, about 9.9 degrees; and after 1000 seconds, about 31.5 degrees. This will be the same whether the sensor is moving, or sitting still. Without some other angle reference, this will be a fundamental uncertainty in the result of the angle calculation.

Notes: Converting Angle Random Walk and PSD/FFT Noise Values

Different manufacturers will quote noise specifications in different ways. Some will quote an angle random walk (ARW); some will quote a PSD or FFT noise density; some will quote a total noise, one or three sigma variation in the output of the sensor. Below are some methods to convert between the various specifications. See IEEE Std. 952-1997 C.1.1 for more complete discussion of this.

PSD ? ARW

$$ARW\left(^\circ/\sqrt{hr}\right)=\frac{1}{60}\sqrt{PSD\left[\left(\frac{^\circ}{hr}\right)^2\Big/Hz\right]}$$

FFT ? ARW

$$ARW\left(^\circ/\sqrt{hr}\right)=\frac{1}{60}FFT\left[\left(\frac{^\circ}{hr}\right)\Big/\sqrt{Hz}\right]$$

s, BW ? ARW

$$ARW\left(^\circ/\sqrt{hr}\right)=\frac{1}{60}s\left(\frac{^\circ}{hr}\right)\cdot\frac{1}{\sqrt{BW(Hz)}},$$

where s is the standard deviation of the signal, and BW is the effective bandwidth of the sensor in Hz. This creates an estimate of the noise density that is then used to estimate the angle random walk.

References

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- See all of Crossbow Technology's products: inertial sensors, accelerometers, magnetometers, and cutting edge wireless networked sensors. Download product data sheets, manuals and software.

IEEE Std 952-1997, "Guide and Test Procedure for Single Axis Interferometric Fiber Optic Gyros," IEEE, 1997, p.63.

- IEEE standard for defining methods to characterize FOG sensors. Includes a long appendix describing noise measurement and analysis in more detail.